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A NEW INTERVAL-VALUED INTUITIONISTIC FUZZY MODEL TO GROUP DECISION MAKING FOR THE SELECTION OF OUTSOURCING PROVIDERS

Abstract. *The real-world economic conditions have inevitably forced many companies to pursue outsourcing as a suitable long term planning tool to reduce operating costs and improve their competitiveness in different marketplaces. One of the critical activities for outsourcing success is the outsourcing provider selection, which may be regarded as a type of multi-criteria decision making (MCDM) problem. In this study, we propose a multiple-criteria group decision making model under interval-valued intuitionistic fuzzy environment to select the best outsourcing provider. First, an IVIF-weighted geometric averaging (IVIFWGA) operator is employed to aggregate all individual IVIF-decision matrices provided by a group of experts into a collective IVIF-decision matrix. Then, a new version of ELECTRE method in an IVIF environment by novel indexes is proposed for the evaluation process in terms of insufficient and inaccurate information. Finally, to demonstrate its usefulness, an application example for evaluating of outsourcing providers is given from the recent literature.*

Keywords: *Multiple criteria group decision making (MCGDM); ELCTERE; Interval-valued intuitionistic fuzzy sets (IVIFSs); Outsourcing.*

JEL Classification: C02, C44, C61, C63, L00

1. Introduction

Realistic decision making plays a vital role in business management success. Making decisions based on scientific methodologies is the main challenge of managers and system experts because it gives the organizations the vital advantages they need to survive in the fierce competition of a global market. Considering qualitative multiple criteria with conflicting nature is often a part of decision making process. Due to complexity and uncertainty of business environment decision should be made based on relevant opinions of experts. For this reason, participation of many experts in the decision making process is inevitable. Thus, employing an appropriate tool that enables faster realistic decision making is necessary (Noor-E-Alam et al., 2011). This is the reason for development of multi-criteria decision making (MCDM) methods. The main objective of MCDM is to establish overall preferences among alternative options. This enables MCDM methods to be applied in outranking alternatives or final decision of choice (Kabak et al., 2012).

The outranking methods as a special subgroup of MCDM meet the particular requirements of these soft decisions through the notion of weak preference and incomparability, which better represent the real decision situations (Geldermann et al., 1996; Spengler et al., 1996). Elimination and choice expressing the reality (ELECTRE) I as the first outranking method was introduced by Roy (1968). ELECTRE as a popular MCDM method has been successfully applied in many real-world situations (Elitzur et al., 2012). Azadnia et al. (2011) used the Fuzzy C-Means (FCM) clustering as a data mining model to cluster suppliers into groups and ELECTRE method had been employed to rank the suppliers. Sevkli (2010) compared and contrasted crisp and fuzzy ELECTRE methods for supplier selection in a real industry case. Teixeira de Almeida (2007) proposed a model that integrates ELECTRE method and utility function for outsourcing contracts selection. The model takes into account multi-criteria evaluation through ELECTRE method. Also, each criterion is evaluated through a utility function.

Classical MCDM methods consider the ratings of alternatives and the weights of criteria as crisp numbers, in spite of the fact that in real life-situations such as engineering, social sciences, medical sciences, and economics this assumption is barely possible. For this reason, to deal with MCDM problems various kinds of membership functions that indicate uncertain factors are applied (Xiao et al., 2012). Atanassov and Gargov (1989) introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFS) as a further generalization of that of intuitionistic fuzzy sets (IFS). The characteristic of the IVIFS is based on the assumption that the values of membership function and non-membership function are expressed by intervals rather than crisp numbers. Recently, Hashemi et al. (2014) proposed an IVIF-multi-attribute group decision making (MAGDM) model based on the concept of the compromise ratio method and modern IVIF sets under the group decision making process. Park et al. (2011) developed TOPSIS method to handling MAGDM problems under an IVIF-environment where the information about attributes weights is partially known. To solve multi-attribute decision making problems Li (2011) extended a nonlinear programming method that was based on closeness coefficient. In his proposed extension, IVIFSs were employed to express ratings of alternatives on attributes and preference information on attributes was incomplete.

A combination of environmental pressure, competitive pressure and efficiency motivates outsourcing (Hsu et al., 2013; Tate et al., 2009). Traditionally, outsourcing is an abbreviation for “outside resource using” (Bühner & Tuschke, 1997; Arnold, 2000). Many firms attempt to enhance competitiveness, reduce costs, and pay attention to internal resources and core activities and hereby further sustain competitive advantages by outsourcing (Elitzur et al., 2012). To be successful in the outsourcing, a company or an organization should have strong relationships with its outsourcing providers. Selecting outsourcing providers should take this factor into consideration. However, the fact remains that choosing suitable outsourcing providers is not an easy job. This difficulty is caused by outsourcing providers inability to meet all selection criteria (or attributes, factors, indexes) at the same time (Kabak et al., 2012). Therefore, selecting outsourcing providers may be regarded as a type of the MCDM problems (Hsu et al., 2013; Ho et al., 2012; Liou et al., 2011, Vahdani et al., 2014).

This paper presents a novel multi-criteria group decision making (MCGDM) model based on ELECTRE method in an IVIF-environment. Characteristics of alternatives and decision criteria are represented by linguistic terms and then are converted into interval-valued intuitionistic fuzzy number (IVIFN). In the proposed ELECTRE method, the calculation process of concordance and discordance dominance matrixes are based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution. Then, the final ranking is calculated according to the relative closeness to the ideal solutions. Furthermore, application example from the recent literature is examined for the outsourcing decisions to demonstrate the implementation process of the IVIF-MCGDM model.

In order to do so, the remainder of this paper is set out as follows. A brief overview of interval-valued intuitionistic fuzzy sets is given in Section 2. The ELECTRE methods based on interval-valued intuitionistic fuzzy and algorithm of proposed model are described in Section 3. A real application from the recent literature is presented in Section 5 to illustrate the steps of the proposed model. In the final section, conclusions are drawn for the research.

2. Basic concepts and operations of interval-valued intuitionistic fuzzy sets

Let a set X be fixed, an IVIFS in X is defined as (Atanassov & Gargov, 1989):

$$\tilde{A} = \left\{ \left\langle x, \bar{\mu}_{\tilde{A}}(x), \bar{\nu}_{\tilde{A}}(x) \right\rangle \mid x \in X \right\} \tag{1}$$

where $\bar{\mu}_{\tilde{A}}(x) \subset [0,1]$, $\bar{\nu}_{\tilde{A}}(x) \subset [0,1]$, $x \in X$ and $\sup \bar{\mu}_{\tilde{A}}(x) + \sup \bar{\nu}_{\tilde{A}}(x) \le 1$, $\forall x \in X$. Especially, if $\inf \bar{\mu}_{\tilde{A}}(x) = \sup \bar{\mu}_{\tilde{A}}(x)$ and $\inf \bar{\nu}_{\tilde{A}}(x) = \sup \bar{\nu}_{\tilde{A}}(x)$, then the IVIFS $\tilde{A}$ is reduced to an IFS.

For convenience, an IVIFS $\tilde{A}$ is denoted by $\left\langle [\mu^L_{\tilde{A}}(x), \mu^U_{\tilde{A}}(x)], [\nu^L_{\tilde{A}}(x), \nu^U_{\tilde{A}}(x)] \right\rangle$, where $[\mu^L_{\tilde{A}}(x), \mu^U_{\tilde{A}}(x)] \subset [0,1], [\nu^L_{\tilde{A}}(x), \nu^U_{\tilde{A}}(x)] \subset [0,1], \mu^U_{\tilde{A}}(x) + \nu^U_{\tilde{A}}(x) \le 1$ and for each element *x* we can calculate the hesitancy degree of an interval-valued intuitionistic fuzzy of $x \in X$ in $\tilde{A}$ defined as follows:

$$\bar{\pi}_{\tilde{A}} = [1 - \mu^U_{\tilde{A}}(x) - \nu^U_{\tilde{A}}(x), 1 - \mu^L_{\tilde{A}}(x) - \nu^L_{\tilde{A}}(x)]. \tag{2}$$

Atanassov and Gargov (1989) and Atanassov (1994) proposed basic IVIFSs operations, which ensured that the operational results were IVIFSs in additions to illustrating their suitability for variables calculations under the IVIF-environment. Motivated by the operations in Atanassov and Gargov (1989), Atanassov (1994), and Xu (2007) defined four operational laws of IVIFNs, which can be employed in this paper, as follows:

Let $\tilde{a} = \left\langle [\mu^L_{\tilde{a}}, \mu^U_{\tilde{a}}], [\nu^L_{\tilde{a}}, \nu^U_{\tilde{a}}] \right\rangle$ and $\tilde{b} = \left\langle [\mu^L_{\tilde{b}}, \mu^U_{\tilde{b}}], [\nu^L_{\tilde{b}}, \nu^U_{\tilde{b}}] \right\rangle$ be any two IVIFNs, then

$$\tilde{a} \oplus \tilde{b} = \left\langle [\mu^L_{\tilde{a}} + \mu^L_{\tilde{b}} - \mu^L_{\tilde{a}} \cdot \mu^L_{\tilde{b}}, \mu^U_{\tilde{a}} + \mu^U_{\tilde{b}} - \mu^U_{\tilde{a}} \cdot \mu^U_{\tilde{b}}], [\nu^U_{\tilde{a}} \cdot \nu^L_{\tilde{b}}, \nu^U_{\tilde{a}} \cdot \nu^U_{\tilde{b}}] \right\rangle, \tag{3}$$

$$\tilde{a} \otimes \tilde{b} = \left\langle [\mu^L_{\tilde{a}} \cdot \mu^L_{\tilde{b}}, \mu^U_{\tilde{a}} \cdot \mu^U_{\tilde{b}}], [\nu^L_{\tilde{a}} + \nu^L_{\tilde{b}} - \nu^L_{\tilde{a}} \cdot \nu^L_{\tilde{b}}, \nu^U_{\tilde{a}} + \nu^U_{\tilde{b}} - \nu^U_{\tilde{a}} \cdot \nu^U_{\tilde{b}}] \right\rangle, \tag{4}$$

$$\lambda \tilde{a} = \left\langle [1 - (1 - \mu^L_{\tilde{a}})^\lambda, 1 - (1 - \mu^U_{\tilde{a}})^\lambda], [(\nu^L_{\tilde{a}})^\lambda, (\nu^U_{\tilde{a}})^\lambda] \right\rangle, \quad \lambda > 0 \tag{5}$$

$$\tilde{a}^\lambda = \left\langle [(\mu^L_{\tilde{a}})^\lambda, (\mu^U_{\tilde{a}})^\lambda], [1 - (1 - \nu^L_{\tilde{a}})^\lambda, 1 - (1 - \nu^U_{\tilde{a}})^\lambda] \right\rangle, \quad \lambda > 0 \tag{6}$$

which can ensure the operational results are also IVIFNs.

Definition 1. Let $\tilde{\alpha}_j$ (*j* = 1, 2, ..., *n*) be a collection of IVIFNs, The geometric aggregation operator of the IVIFNs is computed by (Xu & Chen, 2007):

$$IVIFWGA_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \prod_{j=1}^{n} \tilde{\alpha}_j^{\omega j}$$

$$= \left\langle \left[\prod_{j=1}^{n} \left(\mu_{\tilde{\alpha}_j}^{L}\right)^{\omega j}, \prod_{j=1}^{n} \left(\mu_{\tilde{\alpha}_j}^{U}\right)^{\omega j} \right], \left[1 - \prod_{j=1}^{n} \left(1 - \nu_{\tilde{\alpha}_j}^{L}\right)^{\omega j}, 1 - \prod_{j=1}^{n} \left(1 - \nu_{\tilde{\alpha}_j}^{U}\right)^{\omega j} \right] \right\rangle \tag{7}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{\alpha}_j$ (j = 1, 2, ..., n), $\omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_j = 1$.

Definition 2. Let A and B be two IVIFSs.The distance between A and B can be defined as follows (Park et al., 2011):

$$D(A,B) = \sqrt{\frac{1}{4n} \sum_{j=1}^{n} \left[\left(\mu_A^L(x_j) - \mu_B^L(x_j)\right)^2 + \left(\mu_A^U(x_j) - \mu_B^U(x_j)\right)^2 + \left(\nu_A^L(x_j) - \nu_B^L(x_j)\right)^2 + \left(\nu_A^U(x_j) - \nu_B^U(x_j)\right)^2 \right]} \tag{8}$$

3. ELECTRE methods based on interval-valued intuitionistic fuzzy

In this section, concordance and discordance sets and proposed IVIF ELECTRE method (including the algorithm) are introduced. We will use the IVIF ELECTRE method algorithm to demonstrate numerical examples.

ELECTRE methods are based on binary outranking relations that are established by the decision maker and are not necessarily transitive. Partial ordering of non-dominant alternatives becomes possible by using the relationship. Two distinct subsets are provided to divide each criterion in different alternatives for each pair of alternatives k and l $(k, l = 1, 2, \ldots, m$ and $k \neq 1)$. The concordance set E_{kl} of A_k and A_l includes all criteria, in which A_k is favoured to A_l. To put it differently, $E_{kl} = \{j | x_{kj} \geq x_{lj}\}$, where $J = \{j | j = 1, 2, \ldots, n\}$. $F_{kl} = \{j | x_{kj} < x_{lj}\}$ denotes the complementary subset that is the discordance set. In the introduced IF ELECTRE method, it is possible to classify different types of concordance and discordance sets by applying the concepts of score function, accuracy function, and intuitionistic index, and employ concordance and discordance sets for the purpose of concordance and discordance matrices development, respectively. The concepts of positive and negative ideal points can be used to decide the best alternative.

3.1. Concordance and discordance sets

Applying the concepts of score function, accuracy function, and hesitancy degree of the IVIF values enables comparison of different alternatives to their IVIF values. If alternatives have the same score degree, the preferred alternative gets the higher score degree or higher accuracy degree. A larger membership degree or smaller non-membership degree is reflected by a higher score degree, and a smaller hesitation degree shows higher accuracy degree. Different kinds of concordance sets such as the concordance set, midrange concordance set, and weak concordance set based on the notion of score function and accuracy function are introduced. These sets can also be divided in three groups of the discordance set, midrange discordance set, and weak discordance set.

Consider $\tilde{X} = \left\langle [\mu_{\tilde{X}}^L, \mu_{\tilde{X}}^U], [\nu_{\tilde{X}}^L, \nu_{\tilde{X}}^U] \right\rangle$ as an IVIFN. The concordance set C_{kl} of A_k and A_l includes all criteria in which A_k is more favorable than A_l. In the proposed method classify concordance sets are based on the concepts of score function, accuracy function, and hesitancy degree of the IVIFN. The concordance set C_{kl} is introduced as follows:

$$C_{kl}^1 = \left\{ j \middle| \left(\mu_{kj}^L + \mu_{kj}^U\right) \geq \left(\mu_{lj}^L + \mu_{lj}^U\right), \left(\nu_{kj}^L + \nu_{kj}^U\right) < \left(\nu_{lj}^L + \nu_{lj}^U\right) and \left(\mu_{kj}^L + \mu_{kj}^U + \nu_{kj}^L + \nu_{kj}^U\right) > \left(\mu_{lj}^L + \mu_{lj}^U + \nu_{lj}^L + \nu_{lj}^U\right) \right\} \tag{9}$$

where $J = \left\{ j \middle| j = 1, 2, \ldots, n \right\}$, a larger IVIF value is determined by a larger score, a lower hesitancy degree is reflected by a higher accuracy degree, and Equations (10) or (11) are less concordant than (9).

The following is the definition of midrange concordance set C_{kl}^2.

$$C_{kl}^2 = \left\{ j \middle| \left(\mu_{kj}^L + \mu_{kj}^U\right) \geq \left(\mu_{lj}^L + \mu_{lj}^U\right), \left(\nu_{kj}^L + \nu_{kj}^U\right) < \left(\nu_{lj}^L + \nu_{lj}^U\right) \text{and} \left(\mu_{kj}^L + \mu_{kj}^U + \nu_{kj}^L + \nu_{kj}^U\right) \leq \left(\mu_{lj}^L + \mu_{lj}^U + \nu_{lj}^L + \nu_{lj}^U\right) \right\} \tag{10}$$

The hesitancy degree is the main difference between (9) and (10); the hesitancy degree at the *k*th alternative (candidate) by regarding the *j*th criterion can be higher than the *l*th alternative versus the *j*th criterion in the midrange concordance set. Therefore, Eq. (10) is less concordant than (9).

The definition of the weak concordance set C_{kl}^3 is as follows.

$$C_{kl}^3 = \left\{ j \middle| \left(\mu_{kj}^L + \mu_{kj}^U\right) \geq \left(\mu_{lj}^L + \mu_{lj}^U\right) \text{and} \left(\nu_{kj}^L + \nu_{kj}^U\right) \geq \left(\nu_{lj}^L + \nu_{lj}^U\right) \right\} \tag{11}$$

Non-membership's degree at the *k*th alternative (candidate) by regarding the *j*th criterion can be higher than the *l*th alternative in terms of the *j*th criterion in the weak concordance set; therefore, Eq. (11) is less concordant than (10).

The discordance set includes all criteria, in which A_k is not more favorable than A_l. The discordance set D_{kl}^1 applying the mentioned basis is proposed as follows:

$$D_{kl}^1 = \left\{ j \middle| \left(\mu_{kj}^L + \mu_{kj}^U\right) < \left(\mu_{lj}^L + \mu_{lj}^U\right), \left(\nu_{kj}^L + \nu_{kj}^U\right) \geq \left(\nu_{lj}^L + \nu_{lj}^U\right) \text{and} \left(\mu_{kj}^L + \mu_{kj}^U + \nu_{kj}^L + \nu_{kj}^U\right) \leq \left(\mu_{lj}^L + \mu_{lj}^U + \nu_{lj}^L + \nu_{lj}^U\right) \right\} \tag{12}$$

The formula is also based on the same fact that a larger score means a larger IVIF value and a higher accuracy degree shows a lower hesitancy degree.

The following is the definition of midrange discordance set D_{kl}^2:

$$D_{kl}^{2}=\left\{j\middle|\left(\mu_{kj}^{L}+\mu_{kj}^{U}\right)<\left(\mu_{lj}^{L}+\mu_{lj}^{U}\right),\left(\nu_{kj}^{L}+\nu_{kj}^{U}\right)\geq\left(\nu_{lj}^{L}+\nu_{lj}^{U}\right)\text{and}\left(\mu_{kj}^{L}+\mu_{kj}^{U}+\nu_{kj}^{L}+\nu_{kj}^{U}\right)>\left(\mu_{lj}^{L}+\mu_{lj}^{U}+\nu_{lj}^{L}+\nu_{lj}^{U}\right)\right\} \tag{13}$$

Eq. (13) is less discordant than (12).

The following is the definition of the weak discordance set D_{kl}^{3} :

$$D_{kl}^{3}=\left\{j\middle|\left(\mu_{kj}^{L}+\mu_{kj}^{U}\right)<\left(\mu_{lj}^{L}+\mu_{lj}^{U}\right)\text{and}\left(\nu_{kj}^{L}+\nu_{kj}^{U}\right)<\left(\nu_{lj}^{L}+\nu_{lj}^{U}\right)\right\} \tag{14}$$

Both membership and non-membership' degrees at the *k*th alternative (candidate) by regarding the *j*th criterion can be lower than the *l*th alternative versus the *j*th criterion in the weak discordance set; therefore, Eq. (14) is less discordant than (13).

In this paper, the concept of concordance and discordance sets is applied to calculate concordance and discordance matrices and the proposed IVIF ELECTRE method is applied to determine the aggregate dominance matrix. The best alternative is then selected.

3.2. IVIF ELECTRE method

The proposed IVIF ELECTRE method is an integration of IVIFs and ELECTRE method with the evaluation information. The IVIF ELECTRE method calculates the relative value of the concordance set by the concordance index. The concordance index is measured by calculating sum of the weights associated with related criteria and relations that are included in the concordance sets. Therefore, the concordance index C_{kl} between A_k and A_l in this paper is defined as:

$$\tilde{g}_{kl}=w_{C^1}\times\sum_{j\in C_{kl}^{1}}\tilde{w}_j+w_{C^2}\times\sum_{j\in C_{kl}^{2}}\tilde{w}_j+w_{C^3}\times\sum_{j\in C_{kl}^{3}}\tilde{w}_j \tag{15}$$

where the weights of the concordance, midrange concordance, and weak concordance sets are respectively denoted by w_{C^1} , w_{C^2} and w_{C^3}, and w_j shows criteria weight The relative dominance of a certain alternative over a competing alternative is determined by the concordance index. This index is founded on attaching the relative weight to the successive decision criteria. .

The concordance matrix G is defined as follows:

$$\tilde{G}=\begin{bmatrix} - & \tilde{g}_{12} & \cdots & \cdots & \tilde{g}_{1m} \\ \tilde{g}_{1m} & - & \tilde{g}_{23} & \cdots & \tilde{g}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ \tilde{g}_{(m-1)1} & \cdots & \cdots & - & \tilde{g}_{(m-1)m} \\ \tilde{g}_{m1} & \tilde{g}_{m2} & \cdots & \tilde{g}_{m(m-1)} & - \end{bmatrix} \tag{16}$$

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where g^* and g^- indicates the maximum and the minimum values of $\tilde{g}_{kl}$ .They respectively denote the positive ideal point and negative ideal point. Also, a higher value of g_{kl} determines that A_k is more favorable than A_l and vice versa.

The discordance index is denoted as follows:

$$h_{kl} = \frac{\max_{j \in D_{kl}} w_D^* \times d\left(\tilde{x}_{kj}, \tilde{x}_{lj}\right)}{\max_{j \in J} d\left(\tilde{x}_{kj}, \tilde{x}_{lj}\right)} \tag{17}$$

where Equation (8) determines $d\left(\tilde{x}_{kj}, \tilde{x}_{lj}\right)$, and w_D^* depending on various kinds of discordance sets, is equal to w_{D^1} , w_{D^2} or w_{D^3}. Discordance weight, midrange discordance, and weak discordance sets are respectively included in these sets.

The discordance matrix H is defined as follows:

$$H = \begin{bmatrix} - & h_{12} & \cdots & \cdots & h_{1m} \\ h_{1m} & - & h_{23} & \cdots & h_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ h_{(m-1)1} & \cdots & \cdots & - & h_{(m-1)m} \\ h_{m1} & h_{m2} & \cdots & h_{m(m-1)} & - \end{bmatrix} \tag{18}$$

where h^* and h^- indicate the maximum and the minimum values of h_{kl} , respectively. In addition, the maximum value means the negative ideal point and the minimum value shows the positive ideal points. A lower value of h_{kl} means that A_k is more favorable than A_l and vice versa.

Concordance dominance matrix calculation process is justified on the notion that the preferred alternative should simultaneously be in the shortest distance and the farthest distance from the positive ideal solution and the negative ideal solution, respectively. Therefore, the concordance dominance matrix K is introduced in the following:

$$K = \begin{bmatrix} - & k_{12} & \cdots & \cdots & k_{1m} \\ k_{1m} & - & k_{23} & \cdots & k_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ k_{(m-1)1} & \cdots & \cdots & - & k_{(m-1)m} \\ k_{m1} & k_{m2} & \cdots & k_{m(m-1)} & - \end{bmatrix} \tag{19}$$

where

$$k_{kl} = \frac{d\left(\tilde{g}^*, \tilde{g}_{kl}\right)}{d\left(\tilde{g}^*, \tilde{g}_{kl}\right) + d\left(\tilde{g}^-, \tilde{g}_{kl}\right)}, \tag{20}$$

which refers to relative closeness to the positive ideal point. A_k is less favorable than A_l if value of k_{kl} is higher.

The calculation process of the discordance dominance matrix is justified on the notion that the preferred alternative should be in the farthest distance from the positive ideal solution; therefore, below is the discordance dominance matrix L definition:

$$L = \begin{bmatrix} - & l_{12} & \cdots & \cdots & l_{1m} \\ l_{1m} & - & l_{23} & \cdots & l_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ l_{(m-1)1} & \cdots & \cdots & - & l_{(m-1)m} \\ l_{m1} & l_{m2} & \cdots & l_{m(m-1)} & - \end{bmatrix} \qquad (21)$$

where

$$l_{kl} = \frac{h^* - h_{kl}}{h^* - h^-}, \qquad (22)$$

which refers to relative closeness to the negative ideal point. A_k is preferred to A_l if value of l_{kl} is higher.

The aggregate dominance matrix can measure the distance of each alternative from the ideal positive and negative points. Moreover, it can determine the ranking order of all alternatives. Below is the aggregate dominance matrix *R*:

$$R = \begin{bmatrix} - & r_{12} & \cdots & \cdots & r_{1m} \\ r_{1m} & - & r_{23} & \cdots & r_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ r_{(m-1)1} & \cdots & \cdots & - & r_{(m-1)m} \\ r_{m1} & r_{m2} & \cdots & r_{m(m-1)} & - \end{bmatrix}, \qquad (23)$$

where

$$r_{kl} = \frac{l_{kl}}{k_{kl} + l_{kl}}, \qquad (24)$$

Equations (20) and (22) define k_{kl} and l_{kl}, respectively. r_{kl} denotes the relative closeness to the ideal solution with a range from 0 to 1. A higher value of r_{kl} determines that the alternative A_k is better than the alternative A_l, meaning that in comparison with the other alternative it is at the same time closer to the positive ideal point and farther from the negative ideal point. To choose the best alternative,

$$\overline{T}_k = \frac{1}{m-1} \sum_{l=1, l \neq k}^{m} r_{kl}, \qquad k = 1, 2, \ldots, m, \qquad (25)$$

and $\overline{T}_k$ shows the final evaluation value. All alternatives depending on $\overline{T}_k$ can be ranked. The best alternative A^*, which is also the closest to the positive ideal point and the farthest from the negative ideal point, is generated and defined below:

$$A^* = \max\{\overline{T}_k\}$$

where A^* is the best alternative. The whole introduced IVIF-ELECTRE method algorithm is defined in the following.

3.3. Algorithm

This section describes a new MCGDM approach, the IVIF-ELECTRE method, for decision making by integrating the IVIF and the ELECTRE methods with evaluation information.

For the MCGDM problem, let $E = \{E_1, E_2, \dots, E_l\}$ be the set of the experts or DMs, $A = \{A_1, A_2, \dots, A_m\}$ be a finite set of alternatives, and $C = \{C_1, C_2, \dots, C_n\}$ be the set of conflicting attributes.

The characteristic of the candidate A_i is represented by an IVIFN as follows:

$$\widetilde{A}_i = \{\langle C_j, [\mu^L_{\tilde{A}_i}(C_j), \mu^U_{\tilde{A}_i}(C_j)], [\nu^L_{\tilde{A}_i}(C_j), \nu^U_{\tilde{A}_i}(C_j)]\rangle | C_j \in C\},$$

where $0 \le \mu^U_{\tilde{A}_i}(C_j) + \nu^U_{\tilde{A}_i}(C_j) \le 1,\ \ \mu^L_{\tilde{A}_i}(C_j) \ge 0,\ \ \nu^L_{\tilde{A}_i}(C_j) \ge 0,\ \ j = 1, 2, \dots, n,\ i = 1, 2, \dots, m.$

The IVIFN that is the pair of intervals $\overline{\mu}^{(k)}_{\tilde{A}_i}(C_j) = [a^{(k)}_{ij}, b^{(k)}_{ij}], \overline{\nu}^{(k)}_{\tilde{A}_i}(C_j) = [c^{(k)}_{ij}, d^{(k)}_{ij}]$ for $C_j \in C$ is denoted by $\widetilde{x}^{(k)}_{ij} = \langle [a^{(k)}_{ij}, b^{(k)}_{ij}], [c^{(k)}_{ij}, d^{(k)}_{ij}] \rangle$, where $[a^{(k)}_{ij}, b^{(k)}_{ij}]$ indicates the degree that the candidate A_i satisfies the attribute C_j provided by the expert or DM E_k (k=1, 2,…, l), $[c^{(k)}_{ij}, d^{(k)}_{ij}]$ indicates the degree that the candidate A_i does not satisfies the attribute C_j given by the expert E_k.

$X^{(k)} = \left(\widetilde{x}^{(k)}_{ij}\right)_{m \times n}$ provided by the expert E_k as an IVIF-decision matrix is obtained as the following form:

$$X^{(k)} = \left(\widetilde{x}^{(k)}_{ij}\right)_{m \times n} =$$

$$\begin{array}{c|cccc|}
 & C_1 & C_2 & \dots & C_n \\
A_1 & \langle [a^{(k)}_{11}, b^{(k)}_{11}], [c^{(k)}_{11}, d^{(k)}_{11}] \rangle & \langle [a^{(k)}_{12}, b^{(k)}_{12}], [c^{(k)}_{12}, d^{(k)}_{12}] \rangle & \dots & \langle [a^{(k)}_{1n}, b^{(k)}_{1n}], [c^{(k)}_{1n}, d^{(k)}_{1n}] \rangle \\
 & \langle [a^{(k)}_{21}, b^{(k)}_{21}], [c^{(k)}_{21}, d^{(k)}_{21}] \rangle & \langle [a^{(k)}_{22}, b^{(k)}_{22}], [c^{(k)}_{22}, d^{(k)}_{22}] \rangle & \dots & \langle [a^{(k)}_{2n}, b^{(k)}_{2n}], [c^{(k)}_{2n}, d^{(k)}_{2n}] \rangle \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
A_m & \langle [a^{(k)}_{m1}, b^{(k)}_{m1}], [c^{(k)}_{m1}, d^{(k)}_{m1}] \rangle & \langle [a^{(k)}_{m2}, b^{(k)}_{m2}], [c^{(k)}_{m2}, d^{(k)}_{m2}] \rangle & \dots & \langle [a^{(k)}_{mn}, b^{(k)}_{mn}], [c^{(k)}_{mn}, d^{(k)}_{mn}] \rangle
\end{array}$$

An algorithm and decision process of the IVIF-ELECTRE method can be summarized in the following steps:

Step 1. A committee of the experts or DMs (E_k, k=1,2, …, l) is established to determine the best alternative among a set of potential alternatives (candidates) by considering the conflicting attributes.

Step 2. Proper attributes are identified for the selection problem.

Step 3. The weight of each selected attribute j by kth DM is subjectively described by a linguistic term and is transformed into the IVIFN ($\widetilde{W}_j^{(k)} = \left\langle \bar{\mu}_{\widetilde{W}_j}^{(k)}, \bar{\nu}_{\widetilde{W}_j}^{(k)} \right\rangle = \left\langle [w_{j1}^{(k)}, w_{j2}^{(k)}], [w_{j3}^{(k)}, w_{j4}^{(k)}] \right\rangle$).

Step 4. The performance rating of each potential alternative versus the selected attributes is evaluated by each DM ($\widetilde{x}_{ij}^{(k)}$) and the IVIF-performance matrix is formed for each DM ($X^{(k)}$).

Step 5. The aggregated IVIF-weight of each selected attribute based on the interval-valued intuitionistic fuzzy weighted geometric averaging (IVIFWGA) operator is calculated, $IVIFWGA_\omega(\widetilde{W}_1, \widetilde{W}_2, ..., \widetilde{W}_l)$, by:

$$\widetilde{W}_j = \left\langle \bar{\mu}_{\widetilde{W}_j}, \bar{\nu}_{\widetilde{W}_j} \right\rangle = \left\langle [w_{j1}, w_{j2}], [w_{j3}, w_{j4}] \right\rangle = \left\langle \left[\prod_{k=1}^{l} \left(w_{j1}^{(k)}\right)^{\omega_k}, \prod_{k=1}^{l} \left(w_{j2}^{(k)}\right)^{\omega_k} \right], \left[1 - \prod_{k=1}^{l} \left(1 - w_{j3}^{(k)}\right)^{\omega_k}, 1 - \prod_{k=1}^{l} \left(1 - w_{j4}^{(k)}\right)^{\omega_k} \right] \right\rangle \tag{26}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_l)^T = (1/l, 1/l, ..., 1/l)^T$ is the weight vector of $\widetilde{W}_j$ (j = 1, 2, ..., n), $\omega_k \in$ [0, 1], and $\sum_{k=1}^{l} \omega_k = 1$.

Step 6. The aggregated IVIF-decision matrix is constructed based on opinions of the DMs and the IVIFWGA operator ($IVIFWGA_\omega(\widetilde{x}_{ij}^{(1)}, \widetilde{x}_{ij}^{(2)}, ..., \widetilde{x}_{ij}^{(l)})$) by:

$$\widetilde{x}_{ij} = \left\langle \bar{\mu}_{\widetilde{x}_{ij}}, \bar{\nu}_{\widetilde{x}_{ij}} \right\rangle = \left\langle [a_{ij}, b_{ij}], [c_{ij}, d_{ij}] \right\rangle = \left\langle \left[\prod_{k=1}^{l} \left(a_{ij}^{(k)}\right)^{\omega_k}, \prod_{k=1}^{l} \left(b_{ij}^{(k)}\right)^{\omega_k} \right], \left[1 - \prod_{k=1}^{l} \left(1 - c_{ij}^{(k)}\right)^{\omega_k}, 1 - \prod_{k=1}^{l} \left(1 - d_{ij}^{(k)}\right)^{\omega_k} \right] \right\rangle \tag{27}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_l)^T = (1/l, 1/l, ..., 1/l)^T$ is the weight vector of $\widetilde{x}_{ij}^{(k)}$ (i=1, 2, …, m; j = 1, 2, ..., n; k=1, 2, …, l), $\omega_k \in$ [0, 1], and $\sum_{k=1}^{l} \omega_k = 1$.

Step 7. Identify the concordance and discordance sets. Find C_{kl}^1, C_{kl}^2, C_{kl}^3, D_{kl}^1, D_{kl}^2 and D_{kl}^3 for pair-wise comparisons of alternatives using Eqs (9) to (14).

Step 8.Calculate the concordance matrix G by Eqs (15) and (16).

Step 9.Calculate the discordance matrix H by Eqs (17) and (18).

Step 10.Construct the concordance dominance matrix *K* by Eqs (19) and (20).

Step 11.Construct the discordance dominance matrix *L* by Eqs (21) and (22).

Step 12.Determine the aggregate dominance matrix *R* by Eqs (23) and (24).

Step 13.Calculate the final value of evaluation ($\overline{T}_k$) using Eqs (25) . The alternative with the maximum value is the best alternative. We can rank alternatives in decreasing order.

4. An application for the selection of outsourcing providers

To further demonstrate the proposed model, an application example from Kahraman et al. (2009) is presented for outsourcing provider selection. A committee of five decision makers, DM_1, DM_2, DM_3, DM_4, and DM_5, exists to select the most suitable ISPs. They consider one cost and six benefit criteria in the following:

Price/Cost (C_1)
Product Conformance Quality (C_2)
On-Time Delivery (C_3)
Facility and Technological Capability (C_4)
Quality of Relationship with ISP (C_5)
Professionalism of Salesperson (C_6)
Responsiveness to Customer Needs (C_7).

4.1. Implementation and computational results

Eight potential ISPs, ISP_1, ISP_2, ISP_3, …, ISP_8 are involved for evaluation(Steps 1 and 2). The weights of these eight criteria are obtained by five *DM*s according to linguistic terms described in Table 1 are given in Table 2. Then, the ratings of alternatives with respect to criteria are represented by five *DM*s according to linguistic terms in Table 1areillustrated in Table 3 (Step 3 and 4).

The relative importance of selected criteria provided by all DMs is aggregated by Eq. (26) (Step 5). Also, the ratings of alternatives obtained by all DMs versus each criterion are aggregated by Eq. (27) (Step 6). Table 2 and Table3 is illustrated the aggregated weight of criteria and decision matrix, respectively.

Table 1. Linguistic terms for the rating of weights and alternatives

Linguistic terms	**Interval-valued intuitionistic fuzzy numbers**
Very good (VG)/ Very high (VH)	$\langle[0.90,1.00],[0.00,0.00]\rangle$
Good (G)/high (H)	$\langle[0.70,0.95],[0.00,0.05]\rangle$
Medium good (MG)/medium high (MH)	$\langle[0.50,0.90],[0.05,0.10]\rangle$
Fair (F)/medium (M)	$\langle[0.30,0.70],[0.05,0.25]\rangle$
Medium bad (MB)/medium low (ML)	$\langle[0.10,0.50],[0.20,0.45]\rangle$
Bad (B)/low (L)	$\langle[0.00,0.30],[0.40,0.60]\rangle$
Very bad (VB)/ very low (VL)	$\langle[0.00,0.10],[0.55,0.75]\rangle$

Table 2. The linguistic evaluation of the criteria and their aggregated weights

Critera	**Decision makers**					**Aggregated interval-valued intuitionistic weight**
	DM_1	DM_2	DM_3	DM_4	DM_5	
Price/Cost (C_1)	H	VH	VH	H	H	$\langle[0.77,0.97],[0.00,0.03]\rangle$
Product Conformance Quality (C_2)	VH	VH	H	H	VH	$\langle[0.81,0.98],[0.00,0.02]\rangle$
On-Time Delivery (C_3)	VH	VH	VH	VH	VH	$\langle[0.90,1.00],[0.00,0.00]\rangle$
Facility and Technological Capability (C_4)	H	H	MH	VH	H	$\langle[0.69,0.95],[0.01,0.05]\rangle$
Quality of Relationship with ISP (C_5)	H	VH	VH	H	VH	$\langle[0.81,0.98],[0.00,0.02]\rangle$
Professionalism of Salesperson (C_6)	MH	H	VH	H	VH	$\langle[0.72,0.96],[0.01,0.04]\rangle$
Responsiveness to Customer Needs (C_7)	VH	H	VH	VH	H	$\langle[0.81,0.98],[0.00,0.02]\rangle$

The DMs also give the relative weights as follows:

$$W' = \left[w_{C^1}, w_{C^2}, w_{C^3}, w_{D^1}, w_{D^2}, w_{D^3}\right] == \left[1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}\right]$$

Table 3. Ratings of the Alternatives by the decision makers and their aggregated interval-valued intuitionistic fuzzy decision matrix

Critera	***ISPs***	**Decision makers**					**aggregated values**
		DM_1	DM_2	DM_3	DM_4	DM_5	
Price/Cost (C_1)	ISP_1	VG	G	VG	MG	G	$\langle[0.72,0.96],[0.01,0.04]\rangle$

	ISP_2	B	VB	MB	MB	B	$\langle[0.00,0.30],[0.36,0.59]\rangle$
	ISP_3	MG	G	G	G	MG	$\langle[0.61,0.93],[0.02,0.07]\rangle$
	ISP_4	G	VG	VG	G	G	$\langle[0.77,0.97],[0.00,0.03]\rangle$
	ISP_5	VG	G	G	G	VG	$\langle[0.77,0.97],[0.00,0.03]\rangle$
	ISP_6	MG	MG	MG	MG	MG	$\langle[0.50,0.90],[0.05,0.10]\rangle$
	ISP_7	G	G	G	VG	MG	$\langle[0.69,0.95],[0.01,0.05]\rangle$
	ISP_8	F	F	MB	F	F	$\langle[0.24,0.65],[0.08,0.30]\rangle$
Product Conformance Quality (C_2)	ISP_1	F	MG	F	MG	F	$\langle[0.37,0.77],[0.05,0.19]\rangle$
	ISP_2	G	VG	VG	G	MG	$\langle[0.72,0.96],[0.01,0.04]\rangle$
	ISP_3	VG	G	VG	VG	VG	$\langle[0.86,0.99],[0.00,0.01]\rangle$
	ISP_4	B	MB	F	F	MB	$\langle[0.00,0.52],[0.19,0.42]\rangle$
	ISP_5	G	MG	VG	G	VG	$\langle[0.72,0.96],[0.01,0.04]\rangle$
	ISP_6	G	G	G	G	G	$\langle[0.70,0.95],[0.00,0.05]\rangle$
	ISP_7	MG	MG	G	VG	MG	$\langle[0.60,0.93],[0.03,0.07]\rangle$
	ISP_8	F	F	MG	MG	F	$\langle[0.37,0.77],[0.05,0.19]\rangle$
On-Time Delivery (C_3)	ISP_1	MG	G	G	MG	G	$\langle[0.61,0.93],[0.02,0.07]\rangle$
	ISP_2	B	MB	B	F	B	$\langle[0.00,0.39],[0.30,0.52]\rangle$
	ISP_3	VB	B	VB	B	MB	$\langle[0.00,0.21],[0.43,0.65]\rangle$
	ISP_4	MG	G	MG	MG	G	$\langle[0.57,0.92],[0.03,0.08]\rangle$
	ISP_5	MB	B	F	F	F	$\langle[0.00,0.55],[0.16,0.38]\rangle$
	ISP_6	MG	MG	F	MG	F	$\langle[0.41,0.81],[0.05,0.16]\rangle$
	ISP_7	MG	MG	MG	MG	MG	$\langle[0.50,0.90],[0.05,0.10]\rangle$
	ISP_8	MG	G	MG	G	G	$\langle[0.61,0.93],[0.02,0.07]\rangle$
Facility and Technological Capability (C_4)	ISP_1	F	MG	F	G	MG	$\langle[0.44,0.82],[0.04,0.15]\rangle$
	ISP_2	G	VG	VG	G	VG	$\langle[0.81,0.98],[0.00,0.02]\rangle$
	ISP_3	G	VG	VG	G	VG	$\langle[0.81,0.98],[0.00,0.02]\rangle$
	ISP_4	MG	G	MG	MG	MG	$\langle[0.53,0.91],[0.04,0.09]\rangle$
	ISP_5	B	MB	F	F	MB	$\langle[0.00,0.52],[0.19,0.42]\rangle$
	ISP_6	G	G	G	G	MG	$\langle[0.65,0.94],[0.01,0.06]\rangle$
	ISP_7	MG	G	G	G	G	$\langle[0.65,0.94],[0.01,0.06]\rangle$
	ISP_8	VG	VG	VG	VG	VG	$\langle[0.90,1.00],[0.00,0.00]\rangle$

Quality of Relationship with ISP (C_5)	ISP_1	G	MG	VG	MG	G	$\langle[0.64, 0.94],[0.02, 0.06]\rangle$
	ISP_2	MB	F	F	MB	MB	$\langle[0.16, 0.57],[0.14, 0.38]\rangle$
	ISP_3	F	MG	MG	G	G	$\langle[0.52, 0.87],[0.03, 0.11]\rangle$
	ISP_4	MG	MG	MG	MG	G	$\langle[0.53, 0.91],[0.04, 0.09]\rangle$
	ISP_5	MG	G	G	MG	MG	$\langle[0.57, 0.92],[0.03, 0.08]\rangle$
	ISP_6	F	F	MB	B	F	$\langle[0.00, 0.55],[0.16, 0.38]\rangle$
	ISP_7	F	F	F	F	F	$\langle[0.30, 0.70],[0.05, 0.25]\rangle$
	ISP_8	VG	VG	VG	G	G	$\langle[0.81, 0.98],[0.00, 0.02]\rangle$
Professionalism of Salesperson (C_6)	ISP_1	MG	G	VG	G	MG	$\langle[0.64, 0.94],[0.02, 0.06]\rangle$
	ISP_2	VB	B	F	MB	B	$\langle[0.00, 0.32],[0.34, 0.56]\rangle$
	ISP_3	MG	G	G	G	G	$\langle[0.65, 0.94],[0.01, 0.06]\rangle$
	ISP_4	G	G	G	G	G	$\langle[0.70, 0.95],[0.00, 0.05]\rangle$
	ISP_5	MG	G	G	G	G	$\langle[0.65, 0.94],[0.01, 0.06]\rangle$
	ISP_6	B	MB	MB	F	F	$\langle[0.00, 0.52],[0.19, 0.42]\rangle$
	ISP_7	F	F	MG	MG	F	$\langle[0.37, 0.77],[0.05, 0.19]\rangle$
	ISP_8	G	G	G	VG	VG	$\langle[0.77, 0.97],[0.00, 0.03]\rangle$
Responsiveness to Customer Needs (C_7)	ISP_1	F	MB	B	F	MB	$\langle[0.00, 0.52],[0.19, 0.42]\rangle$
	ISP_2	MG	G	G	MG	MG	$\langle[0.57, 0.92],[0.03, 0.08]\rangle$
	ISP_3	VB	B	MB	B	MB	$\langle[0.00, 0.30],[0.36, 0.59]\rangle$
	ISP_4	G	MG	G	MG	MG	$\langle[0.57, 0.92],[0.03, 0.08]\rangle$
	ISP_5	F	F	F	MB	B	$\langle[0.00, 0.55],[0.16, 0.38]\rangle$
	ISP_6	F	F	F	F	F	$\langle[0.30, 0.70],[0.05, 0.25]\rangle$
	ISP_7	VB	B	F	B	F	$\langle[0.00, 0.34],[0.32, 0.53]\rangle$
	ISP_8	VG	VG	VG	G	G	$\langle[0.81, 0.98],[0.00, 0.02]\rangle$

After calculating the aggregated IF-decision matrix and the weights of eight ISPs, the concordance and discordance sets are identified (Step 7).

Then, the concordance and discordance matrix are calculated (steps 8 and 9). The respective results are as follows:

$$
G=\begin{bmatrix}
- & \langle[0.995,1.000],[0.000,0.000]\rangle & \langle[0.994,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle \\
\langle[0.998,1.000],[0.000,0.000]\rangle & - & \langle[0.994,1.000],[0.000,0.000]\rangle & \langle[0.993,1.000],[0.000,0.000]\rangle \\
\langle[0.996,1.000],[0.000,0.000]\rangle & \langle[0.994,1.000],[0.000,0.000]\rangle & - & \langle[0.987,1.000],[0.000,0.000]\rangle \\
\langle[0.984,1.000],[0.000,0.000]\rangle & \langle[0.997,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle & - \\
\langle[0.983,1.000],[0.000,0.000]\rangle & \langle[0.994,1.000],[0.000,0.000]\rangle & \langle[0.991,1.000],[0.000,0.000]\rangle & \langle[0.965,1.000],[0.000,0.000]\rangle \\
\langle[0.998,1.000],[0.000,0.000]\rangle & \langle[0.958,1.000],[0.000,0.000]\rangle & \langle[0.996,1.000],[0.000,0.000]\rangle & \langle[0.987,1.000],[0.000,0.000]\rangle \\
\langle[0.987,1.000],[0.000,0.000]\rangle & \langle[0.995,1.000],[0.000,0.000]\rangle & \langle[0.967,1.000],[0.000,0.000]\rangle & \langle[0.987,1.000],[0.000,0.000]\rangle \\
\langle[1.000,1.000],[0.000,0.000]\rangle & \langle[1.000,1.000],[0.000,0.000]\rangle & \langle[1.000,1.000],[0.000,0.000]\rangle & \langle[1.000,1.000],[0.000,0.000]\rangle
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
\langle[0.999,1.000],[0.000,0.000]\rangle & \langle[0.995,1.000],[0.000,0.000]\rangle & \langle[0.998,1.000],[0.000,0.000]\rangle & \langle[0.877,1.000],[0.000,0.000]\rangle \\
\langle[0.993,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle & \langle[0.998,1.000],[0.000,0.000]\rangle & \langle[0.958,0.999],[0.000,0.001]\rangle \\
\langle[0.993,1.000],[0.000,0.000]\rangle & \langle[0.997,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle & \langle[0.814,0.980],[0.000,0.020]\rangle \\
\langle[0.998,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle & \langle[0.999,1.000],[0.000,0.000]\rangle & \langle[0.000,0.000],[1.000,1.000]\rangle \\
- & \langle[0.971,1.000],[0.000,0.000]\rangle & \langle[0.997,1.000],[0.000,0.000]\rangle & \langle[0.000,0.000],[1.000,1.000]\rangle \\
\langle[0.999,1.000],[0.000,0.000]\rangle & - & \langle[0.995,1.000],[0.000,0.000]\rangle & \langle[0.814,0.980],[0.000,0.020]\rangle \\
\langle[0.993,1.000],[0.000,0.000]\rangle & \langle[0.997,1.000],[0.000,0.000]\rangle & - & \langle[0.814,0.980],[0.000,0.020]\rangle \\
\langle[1.000,1.000],[0.000,0.000]\rangle & \langle[1.000,1.000],[0.000,0.000]\rangle & \langle[1.000,1.000],[0.000,0.000]\rangle & -
\end{matrix}\right]
$$

$$
H=\begin{bmatrix}
- & 1.000 & 0.480 & 1.000 & 0.542 & 0.478 & 0.666 & 1.000 \\
0.912 & - & 1.000 & 0.921 & 0.887 & 0.748 & 0.753 & 1.000 \\
1.000 & 0.993 & - & 1.000 & 0.329 & 1.000 & 1.000 & 1.000 \\
0.657 & 1.000 & 0.941 & - & 1.000 & 1.000 & 0.843 & 1.000 \\
1.000 & 1.000 & 1.000 & 0.799 & - & 1.000 & 1.000 & 1.000 \\
1.000 & 1.000 & 0.919 & 1.000 & 1.000 & - & 0.853 & 1.000 \\
1.000 & 1.000 & 0.332 & 1.000 & 0.444 & 1.000 & - & 1.000 \\
0.000 & 0.497 & 0.450 & 0.000 & 0.385 & 0.405 & 0.256 & -
\end{bmatrix}
$$

Consequently, the concordance dominance matrix and the discordance dominance matrix are constructed (steps 10 and 11). The respective results are as follows:

$$K=\begin{bmatrix} - & 0.002 & 0.003 & 0.000 & 0.001 & 0.002 & 0.001 & 0.059 \\ 0.001 & - & 0.003 & 0.004 & 0.004 & 0.001 & 0.001 & 0.021 \\ 0.002 & 0.003 & - & 0.006 & 0.004 & 0.001 & 0.000 & 0.090 \\ 0.008 & 0.001 & 0.000 & - & 0.001 & 0.000 & 0.000 & 1.000 \\ 0.008 & 0.003 & 0.004 & 0.017 & - & 0.014 & 0.001 & 0.090 \\ 0.001 & 0.021 & 0.002 & 0.006 & 0.001 & - & 0.003 & 0.090 \\ 0.006 & 0.002 & 0.016 & 0.006 & 0.003 & 0.002 & - & 0.090 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & - \end{bmatrix}$$

$$L=\begin{bmatrix} - & 0.000 & 0.520 & 0.000 & 0.458 & 0.522 & 0.334 & 0.000 \\ 0.088 & - & 0.000 & 0.079 & 0.113 & 0.252 & 0.247 & 0.000 \\ 0.000 & 0.007 & - & 0.000 & 0.671 & 0.000 & 0.000 & 0.000 \\ 0.343 & 0.000 & 0.059 & - & 0.000 & 0.000 & 0.157 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.201 & - & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.081 & 0.000 & 0.000 & - & 0.147 & 0.000 \\ 0.000 & 0.000 & 0.668 & 0.000 & 0.556 & 0.000 & - & 0.000 \\ 1.000 & 0.503 & 0.550 & 1.000 & 0.615 & 0.595 & 0.744 & - \end{bmatrix}$$

The aggregate dominance matrix is determined (step 12).

$$R=\begin{bmatrix} - & 0.000 & 0.994 & 0.000 & 0.999 & 0.995 & 0.998 & 0.000 \\ 0.988 & - & 0.000 & 0.957 & 0.969 & 0.998 & 0.996 & 0.000 \\ 0.000 & 0.688 & - & 0.000 & 0.995 & 0.000 & 0.000 & 0.000 \\ 0.978 & 0.000 & 0.994 & - & 0.000 & 0.000 & 0.998 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.922 & - & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.976 & 0.000 & 0.000 & - & 0.983 & 0.000 \\ 0.000 & 0.000 & 0.977 & 0.000 & 0.994 & 0.000 & - & 0.000 \\ 1.000 & 1.000 & 1.000 & 1.000 & 1.000 & 1.000 & 1.000 & - \end{bmatrix}$$

Finally, the final value of evaluation ($\overline{T}_k$) is calculated (step 13). The obtained results are provided in Table4.The optimal ranking order of alternatives is given by $ISP_8 \succ ISP_2 \succ ISP_1 \succ ISP_4 \succ ISP_7 \succ ISP_6 \succ ISP_3 \succ ISP_5$ and the best alternative is ISP_8.

Table 4. The final value of evaluation $\overline{T}_k$ optimal ranking order of alternatives

Alternatives	$\overline{T}_k$	**Final ranking**
ISP_1	0.570	3
ISP_2	0.701	2
ISP_3	0.240	7
ISP_4	0.424	4
ISP_5	0.132	8
ISP_6	0.280	6
ISP_7	0.282	5
ISP_8	1.000	1

4.2. Discussion

In the this section, to demonstrate the comparison between the proposed outranking method and the compromise solution method by a group of the DMs under uncertainty, the IVIF-TOPSIS method presented by Park et al. (2011) is applied to the application example in the ISP selection. Computational results of the fuzzy TOPSIS are given in Table 5 according to separation measures and the relative closeness coefficient of each alternative in the IVIF-environment.

Table 5. Computational results of the proposed method and IVIF-TOPSIS method

Alternatives	S^*	S^-	C_i	**IVIF-TOPSIS ranking**	**IVIF-ELECTRE ranking**
ISP_1	0.292	0.346	0.542	3	3
ISP_2	0.305	0.360	0.541	4	2
ISP_3	0.364	0.321	0.468	7	7
ISP_4	0.287	0.377	0.568	2	4
ISP_5	0.348	0.297	0.461	8	8
ISP_6	0.309	0.306	0.497	6	6
ISP_7	0.317	0.314	0.498	5	5
ISP_8	0.140	0.456	0.765	1	1

By considering Tables 5, it is found that the rankings of 8 alternatives are similar; in this case, ISP_8 is the first rank and ISP_5 is the eighth rank with respect to seven selected criteria in the *ISP* decision-making problem. A different ranking is found in the second and fourth rankings.

5. Conclusions

This paper has presented a new MCGDM model based on ELECTRE method in an IVIF-environment. In the proposed model, different types of concordance and discordance sets, concordance and discordance set, midrange concordance and discordance set, and weak concordance and discordance set have been classified and presented under IVIF-environment based on the concepts of score function, accuracy function, and intuitionistic index. Also, the calculation process of concordance and discordance dominance matrices is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution. The model has been applied to the real application to the outsourcing provider selection problem from the recent literature. Computational results have illustrated that the alternative ranking order obtained from the proposed model has been confirmed by the intuitionistic fuzzy TOPSIS method for the decision-making problem. The main advantages of the proposed model, compared to the previous methods based on the ELECTRE, are that the concepts of modern fuzzy sets in interval valued environment and ELECTRE method based on shortest distance from the positive ideal solution and farthest distance from the negative ideal solution that are taken into consideration simultaneously. Furthermore, the proposed model has handled the uncertainty through the group decision making process represented by the professional experts. As a direction for future research, the practicality of this paper can be further enhanced through developing the proposed model into a decision support system (DSS) to reduce needed time and effort for computations. In addition, the proposed model can be applied to other decision-making problems in the supply chain management area.

Acknowledgment: ***The first author is grateful for the partially financial support from the Islamic Azad University - Karaj Branch under the research project entitled, "Presenting a group decision-making model for evaluation of outsourcing providers in an intuitionistic fuzzy environment".***

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